Ph.D. Synopsis /work plan

On Topic

Some Aspects of Fixed point theorems

By

Saeed Ahmed Ali Al-Salehi

Yeshwant Mahavidyalaya, Collage,

Swami Ramanand Teerth Marathwada University, Nanded

Supervised by

Dr. V. C. Borkar

Yeshwant Mahavidyalaya, Collage**,**

Swami Ramanand Teerth Marathwada University, Nanded-431606

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Introduction:

Fixed point theory is one of the most powerful and fruitful tools of modern mathematics and may be considered a core subject of nonlinear analysis. In the last 50 years, fixed point theory has been a flourishing area of research form any mathematicians. The origins of the theory, which date to the later part of the nineteenth century, rest in the use of successive approximations to establish the existence and uniqueness of solutions, particularly to differential equations. This method is associated with many celebrated mathematicians like Cauchy, Fredholm, Liouville, Lipschitz, Peano, and Picard. It is worth noting that the abstract formulation of Banach is credited as the starting point to metric fixed point theory. But the theory did not gain enough impetus till Felix Browder's major contribution to the development of the nonlinear functional analysis as an active and vital branch of mathematics.

In a wide range of mathematical, computational, economical, modeling and engineering problems, the existence of a solution to a theoretical or real world problem is equivalent to the existence of a fixed point for a suitable map or operator. Fixed points are therefore of paramount importance in many areas of mathematics, sciences and engineering.

Fixed point theory has the diverse applications in different branches of mathematics, statistics, engineering, and economics in dealing with the problems arising in approximation theory, potential theory, game theory, theory of differential equations, theory of integral equations, and others. Developments in the investigation on fixed points of no expansive mappings, contractive mappings in different spaces like metric spaces, Banach spaces, Fuzzy metric spaces, Cone metric spaces have almost been saturated. After the study of random fixed point theorems initiated by the Prague school of Probability in the 1950's, considerable

attention has been given to the study of random fixed point theorems because of its importance in probabilistic functional analysis and probabilistic models with numerous applications. The introduction of randomness however leads to several new questions of measurability of solutions, probabilistic and statistical aspects of random solutions [11-14]

The theory itself is a beautiful mixture of analysis (pure and applied), topology and geometry. Over the last 60 years or so, the theory of fixed points has been revealed as a very powerful and important tool in the study of nonlinear phenomena. In particular, fixed point techniques have been applied in such diverse fields as biology, chemistry, physics, engineering, game theory and economics.

In numerous cases finding the exact solution is not possible; hence it is necessary to develop appropriate algorithms to approximate the requested result. This is strongly related to control and optimization problems arising in the different sciences and in engineering problems. Many situations in the study of nonlinear equations, calculus of variations, partial differential equations, optimal control and inverse problems can be formulated in terms of fixed-point problems or optimization [15]

In recent years a number of excellent books, monographs, and surveys by distinguished authors about fixed point theory have appeared.

The fixed-point problem (at the basis of the Fixed-Point Theory) may be stated as: Let X be a set, A and B two nonempty subsets of X such that $A \cap B \neq 0$, and $f: A \rightarrow B$ be a map. When does a point $x \in A$ such that $f(x)=x$, also called a fixed point of-f, exist

Fixed Point Theory is divided into three major areas:

- Topological Fixed Point Theory
- Metric Fixed Point Theory
- Discrete Fixed Point Theory

Historically the boundary lines between the three areas were defined by the discovery of three major theorems:

- Brouwer's Fixed Point Theorem [8]
- Banach's Fixed Point Theorem [9]
- Tarski's Fixed Point Theorem [10]

Review of Literature and Development in the subject (Previous work done in the relevant areas):

In 1922, Banach created a famous result called Banach contraction principle in the concept of the fixed point theory [1]. Later, most of the authors intensively introduced many works regarding the fixed-point theory in various of spaces. The concept of a fuzzy metric space was introduced in different ways by some authors (see [2, 3]). Grigori and Sapna [4] introduced the notion of fuzzy contractive mappings and gave some fixed-point theorems for complete fuzzy metric spaces in the sense of George and Veer Amani, and also for Cramoisy and Michalak's fuzzy metric spaces which are complete in Gracie's sense. Mihet [5] developed the class of fuzzy contractive mappings of Grigori and Sapna, considered these mappings in non-Archimedean fuzzy metric spaces in the sense of Kramosil and Michalak, and obtained a fixed- point theorem for fuzzy contractive mappings. At the same time, there are lots of different types of fixed point theorems presented by many authors by expanding the Banach's result in the literature (see [3–7]).etc.

Objective of Research/ proposed hypothesis:

The purpose of this study is to investigate Some Aspects of Fixed-point theorems, The Banach contraction principle, Ordinary differential equations in Banach spaces, and semi linear equations of evolution.

Besides, the study seeks to investigate the application of Fixed-point theorems in the calculations of areas of some geometric shapes.

Methodology to be adopted:

We use different techniques to obtain the results such as:

1) The Banach contraction principle.
2) Fixed point theorems.
3) Fixed points of non-expansive maps.
4) Numerical techniques.
5) Banach fixed-point theorem etc.

Importance of Study/ society applications:

International Status:

Many mathematicians are working in this field, some of them are Yancheng Wang (China), Joy C. Umeda (Nigeria), Hayle N. Saleh (Yemen), C.E. Chime (Nigeria), Merimee El Harrah (Morocco), Lateef Olakunle Jola Oso (South Africa), Sami Shukri (Jordan), Outgas Rida (Morocco), Kazimierz Wodraska (Poland) Shehnai Yan (Japan), Charles E. Chidume (Nigeria), etc.

National Status:

Some of the Indian Mathematicians are also working on same line. Few of them are G.N.V. Kishore (Bhagavatam), Waleed M. Alfaqih (Aligarh), Shamshad Husain (Aligarh), Mallikarjun (Coimbatore, Tamandu), Shamshad Husain (Aligarh), GNV Kishore (Andhra Pradesh), Md Ahmadullah (Aligarh),
Dakjum Esha (Arunachal Pradesh), Vishal Gupta (Haryana), etc.

Significance of study:

Fixed point theory is a fascinating subject, with an enormous number of applications in various fields of mathematics.

One of the most practical areas of Mathematics is the theory of differential equations.

It plays a vital role in Physics and Engineering. Fixed point theory guarantees that these equations have solutions.

Proposed Work plan /Formulation and Structure of study:

Sr.N	Period	Plan of the work	Targets to be achieved
1	First year	Study of the background required for the research topic, like fixed point theorems and Fixed points of non-expansive maps etc.	Good command on the subject and clear the course work.
2	Second year	Collecting the papers and analyze them.	Try to get some new result based on the previous work.
3	Third Year	Work on some problems to be included in the thesis.	Submit at least three papers.
4	Fourth Year	Publishing some papers	Some more papers and Thesis submission.

References:

[1] S. Banach, “Sur les op´ erations dans les ensembles abstracts et leer application aux ´ equations int´ ergates,” *Fundamental Mathematica*, vol. 3, pp. 133–181, 1922.

[2] Z. Deng, “Fuzzy pseudo-metric spaces,” *Journal of Mathematical Analysis and Applications*, vol. 86, no. 1, pp. 74–95, 1982.

[3] I. Cramoisy and J. Michalak, “Fuzzy metrics and statistical metric spaces,” *Cybernetical*, vol. 11, no. 5, pp. 326–334, 1975.

[4] V. Grigori and A. Sapna, “On fixed-point theorems in fuzzy metric spaces,” *Fuzzy Sets and Systems*, vol. 125, no. 2, pp. 245–252, 2002.

[5] S. Chauhan, S. Bhatnagar, and S. Radanovich’, “Common fixed point theorems for weakly compatible mappings in fuzzy metric spaces,” *Le Mathematise*, vol. 68, no. 1, pp. 87–98, 2013.

[6] D. Mihet, “A class of contractions in fuzzy metric spaces,” *Fuzzy Sets and Systems*, vol. 161, no. 8, pp. 1131–1137, 2010.

[7] P. Salami, C. Vetra, and P. Vetra, “Some new fixed-point results in non-Archimedean fuzzy metric spaces,” *Nonlinear Analysis: Modelling and Control*, vol. 18, no. 3, pp. 344–358, 2013.

[8] Sadeghi, S., Shockley, N., Domenici, T., & Radanovich, S. (2018). Suzuki-type of common fixed-point theorems in fuzzy metric spaces. Math. Slovak, 68(2), 451-462.

[9] Banach, Stefan. "Sur les operations dans les ensembles abstracts et leer application aux equations integrals." Fund. math 3.1 (1922): 133-181.21. Brouwer, L.E.J.: Uber Abidingness von Manning fastigiate. Math. Ann.71, 97–115 (1912)

[10] Tarski, A.: lattice theoretical fixed -point theorem and its applications. Pac. J. Math.5,285–309 (1955)

[11] Seidler, E.: Nonlinear Functional Analysis and its Applications I: Fixed-Point Theorems. Springer, New York, Berlin, Heidelberg, Tokyo (1986

[12] Pinot, J.P.: Fixed- point theorem without convexity. Bull. Soc. Math. France Mem.60,129–152 (1979)

[13]A age, C. T. and Saluki, J. N. On common fixed points for contractive type mappings in cone

metric spaces, Bull. Math. Anal. Appl. 1 (3), 10–15, 2009.
[14] Berended, V. Iterative approximation of fixed points (Springer-Verlag, Berlin, 2007).
[15] Berended, V. Approximating common fixed points of noncommitting almost contractions in metric spaces, Fixed Point Theory 11 (2), 179–188, 2010.

Research Student: **Research Guide:**

Mr. Saeed Ahmed Ali Al-Salehi Dr. V. C. Borkar